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ON THE DISTRIBUTION OF SPACE VELOCITIES OF B AND F TYPE STARS

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1. Introduction

One of the most important problems of the stellar statistics is the determination of the frequency-function of space-velocities for different physical types of the stars.

Schwarzschild has supposed, that this frequency-function has the form:

$$\psi(\xi, \eta, \zeta) = Ae^{-Q(\xi, \eta, \zeta)}, \quad (1)$$

where $Q(\xi, \eta, \zeta)$ is a positively definite quadratic form in respect to the variables ξ, η, ζ . The main purpose of the former studies of statistics of the velocities of the stars was the determination of the coefficients entering in $Q(\xi, \eta, \zeta)$. These coefficients are usually called — „the elements of the velocity ellipsoid“. However there are serious indications that the frequency-function of velocities $\psi(\xi, \eta, \zeta)$ has noticeable deviations from the standard form (1). This was proved for example by Oort's well known investigation on the high velocity stars. In fact, according to Oort the relative number of stars with the velocities > 60 km/sec far exceeds the value one can obtain from (1), when the coefficients of $Q(\xi, \eta, \zeta)$ are determined in the usual way ¹.

The deviations are so strong, that many writers have considered the high velocity stars as a separate group and have determined the different cinematical properties of this group (the velocity of the Sun relative to this group, the dispersion of velocities etc.).

As far as we know there was made no attempt to determine the frequency function of space velocities of the stars without making any assumption on the form of this function. It seems to us that this problem is more interesting, due to the fact that the comparison of the frequency function of velocities of different physical types of the stars may give us the possibility of making some conclusions on the course of stellar evolution. Thus the exceptional dispersion of velocities of cluster-type variables has led us to the conclusion that their mean lifetime is of the same order of magnitude, as the age of our galaxy ².

The direct method of the determination of the frequency function of space velocities requires the knowledge of the space velocity of a great number of stars. The derivation of the space velocity of a given star is possible only in the case when three different quantities have been measured: the radial velocity, the proper motion and the parallax. For some important groups of stars (for example B-type stars) we have very few reliable parallaxes. Therefore the applicability of the direct method is strongly restricted.

¹ Oort: Publications of the Kapteyn Astr. Lab. at Groningen, № 40, 1926.

² V. Ambarzumian: Observatory, 1935, May.

On the other hand the modern catalogues of radial velocities contain very reliable and sufficiently uniform data. Therefore it would be interesting to try to find the frequency function from the radial-velocities only. For some physical types of the stars (for example the B-type stars) this way is unavoidable owing to the lack of the reliable parallaxes. In a recent paper ¹ the writer has given general solution of this problem. It was shown that it is possible to express the solution in the finite form. If we want to find only the distribution of projections of velocity-vectors on the galactic plane (plane problem), then it is necessary to know the radial velocities of the stars situated in some narrow zone near the galactic aequator.

In the present paper we try to find the distribution of velocity-projections on the galactic plane i. e. the distribution of the components ξ and η for some types of stars. Let the corresponding frequency function be $\varphi(\xi, \eta)$. It is clear that:

$$\varphi(\xi, \eta) = \int_{-\infty}^{\infty} \psi(\xi, \eta, \zeta) d\zeta.$$

It seems that on the present stage of the development of stellar statistics it would be difficult to determine the whole function $\psi(\xi, \eta, \zeta)$. At the present extent of observational data it seems more appropriate to try to determine approximately the frequency-function $\varphi(\xi, \eta)$ of the galactic projections and the frequency-function $f(\zeta)$ of the ζ -components of the velocities:

$$f(\zeta) = \int_{-\infty}^{\infty} \int \psi(\xi, \eta, \zeta) d\xi d\eta.$$

Though the knowledge of the functions $\varphi(\xi, \eta)$ and $f(\zeta)$ cannot replace the knowledge of the whole frequency-function $\psi(\xi, \eta, \zeta)$, still they give more information on the behaviour of this frequency-function than the constants of the ellipsoid of dispersion.

2. The fundamental formulae

In the paper cited above the writer has shown that the function $\varphi(\xi, \eta)$ can be determined from the radial velocities of the stars situated in narrow galactic belt. The problem was reduced to the solution of the integral equation:

$$F(V, \alpha) = \int_{(L)} \varphi(\xi, \eta) ds,$$

where $\varphi(\xi, \eta)$ is the unknown function, the integration is carried over the straight line (L) given by the equation:

$$\xi \cos \alpha + \eta \sin \alpha = V$$

and ds is the element of this straight line.

The function $F(V, \alpha)$ is defined by

$$F(V, \alpha) = \frac{f(V, \alpha)}{\int_{-\infty}^{\infty} f(V, \alpha) dV}, \quad (2)$$

¹ V. Ambarzumian: M. N., 96, 172, 1936.

where $f(V, \alpha) dV d\alpha$ is the number of stars observed in the galactic longitudes confined between α and $\alpha + d\alpha$ and with the radial velocities between V and $V + dV$.

As we have shown the solution of the equation (1) has the form:

$$\varphi(\xi, \eta) = -\frac{1}{\pi} \int_0^{\infty} \frac{1}{W} \frac{d\bar{F}(\xi, \eta, W)}{dW} dW, \quad (3)$$

where the function $\bar{F}(\xi, \eta, W)$ is to be obtained from $F(V, \alpha)$ according to the formula:

$$\bar{F}(\xi, \eta, W) = \frac{1}{2\pi} \int_0^{2\pi} F(\xi \cos \alpha + \eta \sin \alpha + W, \alpha) d\alpha. \quad (4)$$

The equation (4) may be written also in the form:

$$\bar{F}(\xi, \eta, W) = \int_{(M)} F(V, \alpha) d\alpha,$$

where the integration is carried over the sinusoid (M)

$$V = W + \xi \cos \alpha + \eta \sin \alpha. \quad (M)$$

We may introduce the absolute value of the velocity U and the direction β instead of the rectangular components ξ and η . Then the equation of the curve (M) takes the form

$$V = W + U \cos(\alpha - \beta)$$

and \bar{F} and φ will depend on U and β instead of ξ and η .

3. The choice of the system of reference

If we do not introduce the corrections depending on the solar motion the arguments ξ and η of the obtained frequency-function $\varphi(\xi, \eta)$ are the components of the velocities of stars referred to the Sun. But it is also possible to introduce according to the usual method the corrections of the observed radial velocities $\Delta V = V_{\odot} \cos(\alpha - \alpha_{\odot})$ for the motion of the Sun in respect to some definite coordinate system. In this case we obtain the frequency-function depending upon the components in that system. It seems that both methods would give the identical results. However, when the function $F(V, \alpha)$ is given numerically and all calculation have a numerical character, one may prefer one of these methods. In the present paper we have supposed that the Sun is fixed and no corrections to the radial velocities have been introduced. Thus we shall obtain the frequency-function of velocities referred to the Sun.

4. The radial velocity-data

The radial velocities used here are taken from the card catalogue composed at the Astronomical Observatory of the Leningrad University. This catalogue contains all stars of the Moore's General Catalogue of the radial velocities ¹ as well as the stars of the Simeis Catalogue ². The galactic zone would be sufficiently wide, since else

² Lick Observatory Publications, 18, 1932.

¹ Publ. de l'Obs. Poulkovo, 43, 1933.

the number of the stars with the observed radial velocities would be insufficient for our purposes. However, we cannot go too far from the galactic aequator, because in high latitudes the influence of ζ component on the radial velocity would be large. We have chosen the galactic belt within the latitudes $b = -20^\circ$ and $b = +20^\circ$.

This paper contains the results of application of our method to three groups of stars.

1) Those stars of $B_0 - B_3$ type from the above mentioned card catalogue, for which $|b| < 20^\circ$. All stars of this type fainter than the magnitude 7.0 were rejected, since for these faint stars the effect of galactic rotation is sufficiently large. At the same time the exact distances of these stars are unknown and therefore it is difficult to introduce the corresponding corrections.

2) The stars of $B_5 - B_9$ type brighter than $7^m.0$ in the same galactic zone.

3) All stars of F-type in the same zone without the restrictions in absolute magnitude (giants and dwarfs together).

In all cases the stars with variable radial velocity were included, when the velocities of the centre of gravity have been determined. In the opposite case the stars with variable velocity were rejected. The number of stars of each group was after this:

Spectral type	Number of stars
$B_0 - B_3$	516
$B_5 - B_9$	398
$F_0 - F_8$	379

5. The table of function $F(V, \alpha)$

The stars of each type were divided into 24 groups according to the galactic longitude. In the first group were included all stars with the longitudes between 0° and 30° , in the second with the longitudes between 15° and 45° etc. Thus each star was entering in two groups. Each of these groups was divided into 12 subgroups according to their radial-velocities. Into first subgroup of each group were included all stars of the group with the radial velocities < -50 km/sec, into the second all stars with the velocities between -50 and -40 km/sec, into the third all stars with the velocities between -40 km/sec and -30 km/sec and into the last group all stars with velocities larger than $+50$ km/sec.

After this the groups situated in the opposite parts of the sky (differing on 180° in longitude) were combined together in such way that each subgroup corresponding to some interval of the positive radial velocities was added to the subgroup corresponding to the corresponding interval of negative radial velocities of another group.

Thus, for example, the stars situated within 210° and 240° of galactic longitude and having the velocities confined within -20 km/sec and -30 km/sec were added to the stars situated between 30° and 60° of galactic longitude, having the radial velocities confined between $+20$ km/sec and $+30$ km/sec.

The reason for such combination lies in the fact that a star seen in the longitude α and having the radial velocity V is for our purposes equivalent to the star with the galactic longitude $\alpha + 180^\circ$ and the radial velocity $-V$.

Therefore we obtained only 12 new groups of stars for the intervals: 0—30°, 15—45°, ... 165—195°, and each of these intervals was divided into 12 subgroups, according to the value of the radial velocities.

The number of stars in each subgroup may be taken as the value of the function $f(V, \alpha)$ for the middle of the corresponding interval of α and in the corresponding interval of V . More exactly, the obtained numbers are the values of $f(V, \alpha) dV d\alpha$. But the product $dV d\alpha$ in all cases remains constant ($dV = 10$ km/sec, $d\alpha = \frac{\pi}{6}$). Therefore our numbers represent the values of $f(V, \alpha)$ multiplied by some unimportant constant factor. Further, the whole number of stars in the group was taken as the value of $\int_{-\infty}^{\infty} f(V, \alpha) dV$ since the groups were formed according to galactic longitudes of the stars.

After this it was easy to determine the values of the function $F(V, \alpha)$ according (2), for all necessary parts of the V, α plane.

The function $F(V, \alpha)$ for our three types of stars, computed in such manner is given in Table 1, 2 and 3.

Table 1
The values of the function $F(V, \alpha)$ for B₀ — B₃ stars

$V \backslash \alpha$	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°
> + 50 km/sec	0.9	—	—	—	—	—	—	1.2	1.1	1.2	1.5	0.9
+ 40 to + 50	0.9	—	—	—	—	—	1.0	1.2	1.1	2.4	3.0	1.7
+ 30 to + 40	8.9	—	—	—	—	—	1.0	1.2	1.1	1.2	6.0	11.9
+ 20 to + 30	24.9	1.2	0.9	—	—	0.9	3.0	3.7	7.7	9.7	13.6	29.8
+ 10 to + 20	17.7	1.2	1.8	3.3	2.5	2.6	6.1	9.8	18.8	20.6	24.1	26.3
0 to + 10	2.7	4.8	4.5	4.1	5.9	6.1	10.1	18.4	17.6	18.2	19.6	6.8
— 10 to 0	4.4	10.9	18.7	25.6	21.9	19.1	20.2	22.2	26.5	23.0	10.6	3.4
— 20 to — 10	8.0	21.8	23.9	25.6	33.7	30.4	19.2	17.2	8.8	8.5	6.0	1.7
— 30 to — 20	10.6	27.9	23.9	19.8	17.7	18.2	11.1	3.7	3.3	1.2	—	1.7
— 40 to — 30	5.3	19.3	7.1	4.1	3.4	4.3	4.0	1.2	—	—	1.5	0.9
— 50 to — 40	0.9	4.8	5.3	3.3	0.8	0.9	6.1	6.2	—	—	—	—
< — 50	0.9	—	—	—	—	3.5	4.0	—	—	—	—	0.9

Table 2
The values of the function $F(V, \alpha)$ for B₅ — B₉ stars

$V \backslash \alpha$	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°
> + 50 km/sec	—	—	—	—	—	—	—	—	—	—	—	—
+ 40 to + 50	—	—	—	—	—	—	—	—	2.4	2.9	—	—
+ 30 to + 40	4.6	—	—	—	—	—	—	1.0	1.2	4.3	8.1	9.1
+ 20 to + 30	6.9	—	0.9	0.8	0.8	2.3	2.5	2.0	1.2	5.8	10.8	13.0
+ 10 to + 20	18.3	1.1	0.9	0.8	0.8	—	2.5	6.8	5.9	14.4	20.3	28.6
0 to + 10	2.3	2.2	1.8	3.9	7.2	7.7	5.7	13.6	24.9	23.1	19.0	10.4
— 10 to 0	9.2	14.1	8.9	8.5	10.4	16.2	23.8	22.4	16.6	11.5	5.4	2.6
— 20 to — 10	9.2	19.6	23.9	18.5	15.2	20.0	14.7	10.7	10.7	2.9	1.4	1.3
— 30 to — 20	11.4	15.2	16.8	26.2	24.1	13.9	9.0	4.8	2.4	1.4	1.4	1.3
— 40 to — 30	4.6	8.7	7.1	4.6	3.2	5.4	4.9	2.0	1.2	—	—	—
— 50 to — 40	—	5.4	6.2	2.3	0.8	0.8	1.0	—	—	—	—	—
< — 50	—	—	0.8	0.8	—	1.6	2.0	—	—	—	—	—

Table 3

The values of the function $F(V, \alpha)$ for F-type stars

$V \backslash \alpha$	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°
< -50 km/sec	—	3.7	6.6	3.8	—	—	1.0	2.5	1.3	1.0	0.9	—
-50 to -40	0.9	4.6	2.8	5.7	5.0	3.3	3.1	2.5	—	—	—	—
-40 to -30	0.9	7.4	7.5	5.7	7.0	2.2	2.1	2.5	2.7	2.1	0.9	0.8
-30 to -20	2.7	7.4	12.3	15.3	11.0	7.6	8.3	3.8	4.0	2.1	0.9	3.3
-20 to -10	8.2	9.3	8.5	13.4	19.1	14.2	6.2	3.8	1.3	3.2	4.4	7.4
-10 to 0	8.2	13.0	10.4	3.8	5.0	13.1	13.5	11.4	12.1	8.4	7.0	8.2
0 to $+10$	11.0	10.2	11.3	12.4	12.0	14.2	15.5	18.9	20.2	13.7	13.2	10.7
$+10$ to $+20$	10.1	4.6	1.9	2.9	4.0	7.6	8.3	6.3	9.4	9.5	6.1	5.7
$+20$ to $+30$	9.2	0.9	0.9	—	—	—	1.0	7.6	10.8	9.5	7.9	9.8
$+30$ to $+40$	3.7	0.9	0.9	—	—	1.1	2.1	1.3	1.3	9.5	15.8	10.7
$+40$ to $+50$	7.3	0.9	—	—	—	—	—	—	—	2.1	3.5	5.7
$> +50$	0.9	—	—	—	—	—	2.1	2.5	—	2.1	2.6	0.8

6. The computation of the function $\overline{F}(\xi, \eta, W)$

The most laborious part of the work was the calculation of the function $\overline{F}(\xi, \eta, W)$, which depends upon three arguments. According to (4)

$$\overline{F}(\xi, \eta, W) = \frac{1}{2\pi} \int_0^{2\pi} F(\xi \cos \alpha + \eta \sin \alpha + W, \alpha) d\alpha, \quad (4)$$

where F is a function of two arguments, tabulated in the table 1, 2 and 3. As it was mentioned above we may introduce the velocity U and its direction α ($\xi = U \cos \alpha$, $\eta = U \sin \alpha$). In this case the equation (4) takes the form:

$$\overline{F}(U, \beta, W) = \int_{(M')} F(V, \alpha) d\alpha \quad (5)$$

where the path of integration on the V, α plane is given by the equation

$$V = W + U \cos(\alpha - \beta). \quad (M')$$

The computation of the integral (5) for different values of U, β, W were made in the following simple way:

The values of the function $F(V, \alpha)$ were plotted on the V, α plane over every rectangle with the dimensions $15^\circ \times 10$ km/sec. After this, on a sheet of tracing paper the curves $V = U \cos \alpha$ were plotted for six different values of the amplitude U ($U = 0$ km/sec, 10 km/sec, . . . 50 km/sec). The superposition of this sheet on our graph and shifting of it along the V axis on W and along the α axis on β gave us the possibility to read immediately through the tracing paper the values of the function $F(W + U \cos(\alpha - \beta), \alpha)$ for the different values of α . The summation of the values of this function taken at 24 equidistant values of α gives the function $F(U, \beta, W)$. Such summations were made for all above mentioned values of U , for

$\beta = 0^\circ, 15^\circ, 30^\circ \dots$, and $W = 0$ km/sec, 10 km/sec . . . 50 km/sec. In the case $U = 0$ we have the straight line instead of sine curve. This straight line remains unchanged after shifting along the α axis. Therefore the number of triples of values of variables U, β and W for which the function $\bar{F}(U, \beta, W)$ has been calculated was equal 726.

7. The computation of the frequency-function

The expression (3) for the frequency-function may be transformed to the form:

$$\varphi(\xi, \eta) = \frac{1}{\pi} \int_0^\infty \frac{\bar{F}(\xi, \eta, 0) - \bar{F}(\xi, \eta, W)}{W^2} dW. \quad (6)$$

We meet some difficulties, when we try to compute numerically this integral. The integrand is indefinite at $W = 0$. However, it is possible to show that the expression

$$\frac{\bar{F}(\xi, \eta, 0) - \bar{F}(\xi, \eta, W)}{W^2}$$

has some definite and finite limit, when $W \rightarrow 0$. If L is this limit, we may write approximately for small values of W

$$\frac{\bar{F}(\xi, \eta, 0) - \bar{F}(\xi, \eta, W)}{W^2} = L.$$

Let us represent our integral in the form:

$$\int_0^\infty = \int_0^{W_1} + \int_{W_1}^\infty,$$

where W_1 is some low velocity. Then we may write simply

$$\int_0^{W_1} = LW_1.$$

The second part of our integral may be calculated according to the usual formulae of numerical integration. The value of L is given by:

$$L = \frac{\bar{F}(\xi, \eta, 0) - \bar{F}(\xi, \eta, W_1)}{W_1^2}.$$

In our calculations we have put $W_1 = 10$ km/sec. As the result of our calculation we have obtained the function $\varphi(\xi, \eta)$ or more exactly $\varphi(U, \beta)$ for the different values of U (0 km, 10 km, . . . 50 km) and β ($0^\circ, 15^\circ, 30^\circ \dots$).

8. The discussion of the results

The graphs 1, 2 and 3 represent the velocity plane where the values of the function $\varphi(U, \beta)$, computed for our three spectral classes are given. On these graphs the centrum corresponds to the stars fixed relative to our Sun.

a) The B-type Stars

One may see from the fig. 1 and 2, that maximum of density in the velocity-plane occurs at $U=20$ km/sec $\beta=195^\circ-210^\circ$ i. e. that Sun has the velocity 20 km/sec in the direction $\beta=15^\circ-30^\circ$ relative to the stars forming this density maximum. The equidensity lines have somewhat elongated form. The direction of this

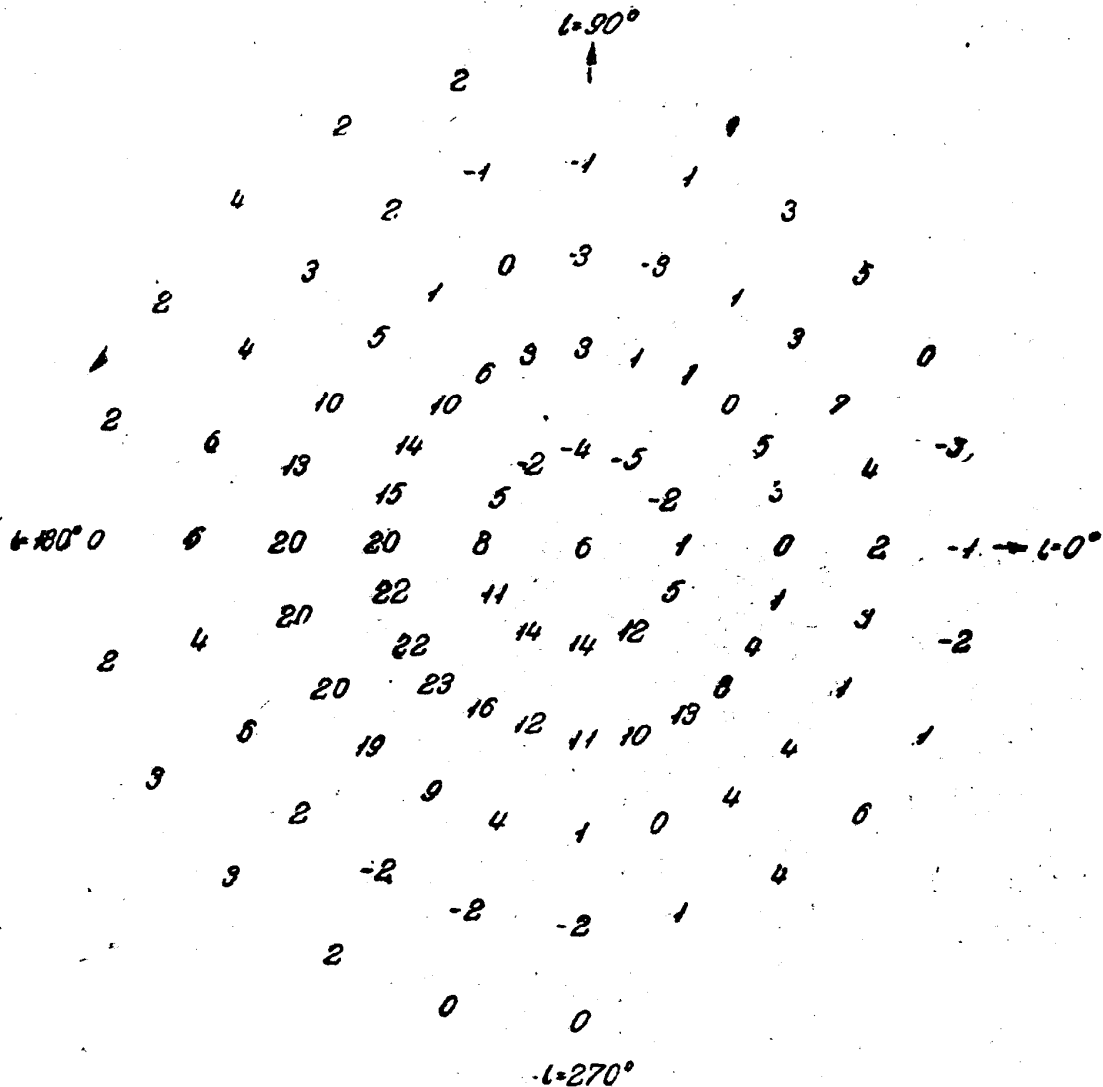


Fig. 1. The velocity distribution of B_0-B_9 stars (from observed radial velocities).
Scale 1 cm = 10 km/sec.

elongation coincides approximately with the known direction of the major axis of the velocity ellipsoid. As it was expected there appear now and there the negative value of $\varphi(U, \beta)$. They are caused by statistical fluctuation. Thus the agreement with the usual methods in the case of B-type stars is satisfactory.

b) The F-type stars

In this case the general character of distribution remains the same (fig. 3).

We have one maximum of density at $U=30$ km/sec, $\beta=195^\circ$. This large value for the velocity of Sun is caused by high percentage of F-dwarfs. There is however another unexpected maximum at $U=10$ km/sec, $\beta=90^\circ$. This means that there

exists a considerable number of stars moving with low velocity (10 km/sec) relative to the Sun. We were at first inclined to suppose that here we have met with the case, where the method fails. However it is easy to show that this additional maximum of density is connected with very peculiar distribution of the radial velocities of the F-type stars.

In fact, among the F-type stars there is a considerable excess of stars with low radial velocities. For example in the Table 4 are given the weighted numbers of stars

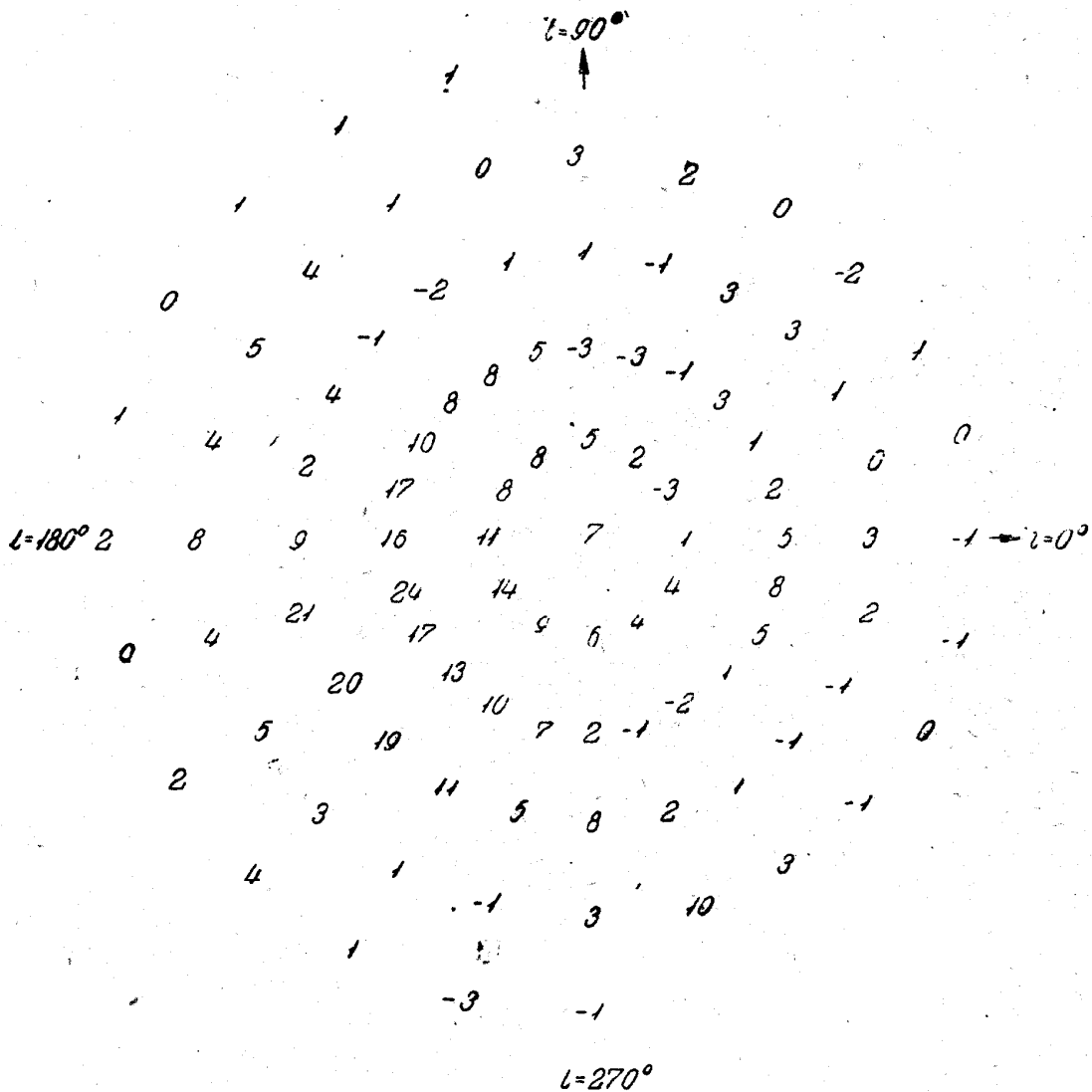


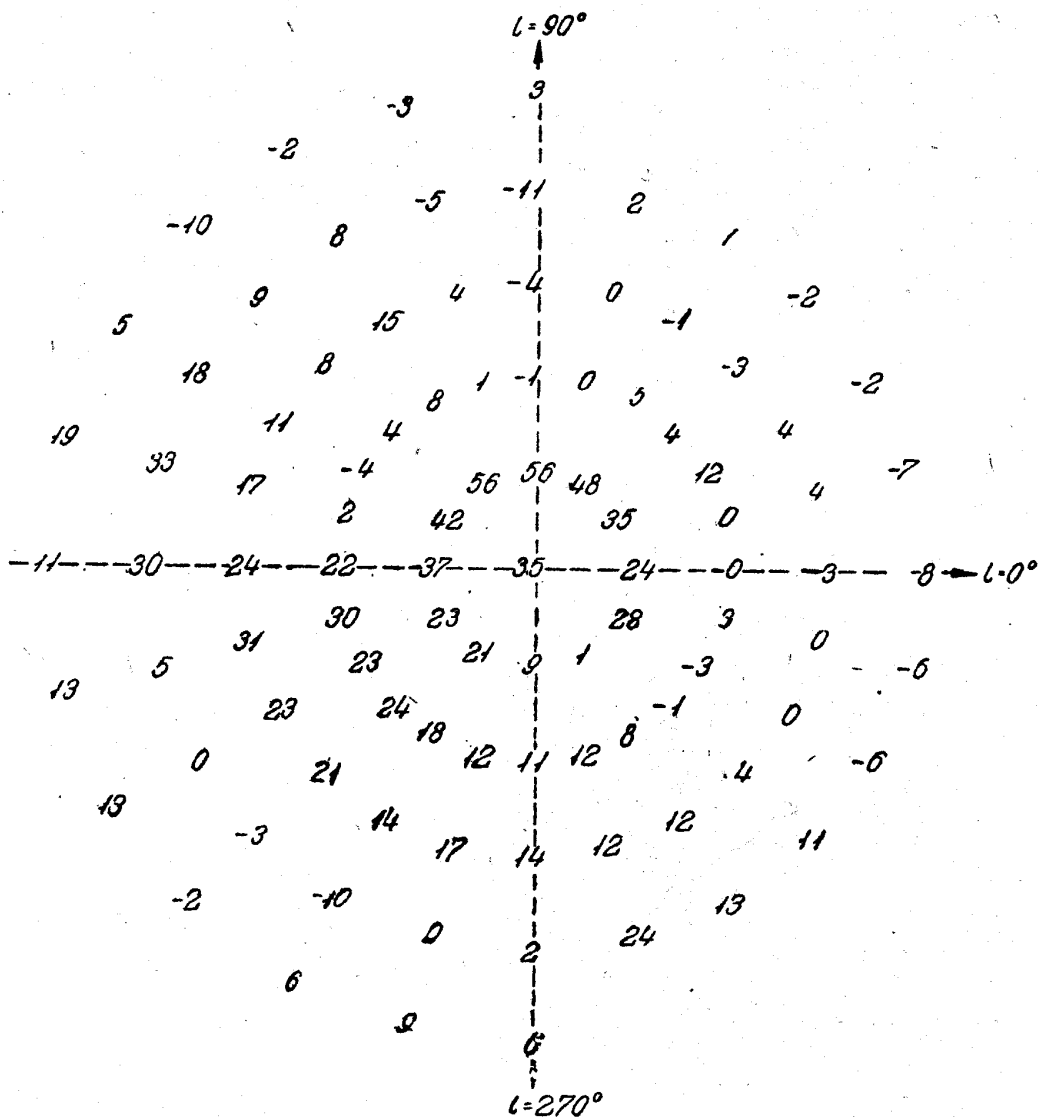
Fig. 2. The velocity distribution of B₅—B₉ stars (from observed radial velocities).
Scale 1 cm = 10 km/sec.

with velocities confined within the limits: 0—10 km/sec, 10—20 km/sec etc., for F-type stars and B stars.

The weight of a single star has been supposed equal to the inverse value of the whole number of stars in the above mentioned longitude-groups. The weighing was necessary in view of nonuniform distribution of the observed stars.

Such an excess of stars with low radial velocity cannot be explained by any kind of statistical fluctuation. It is more probable that it is caused by some real peculiarities in the velocity distribution of F-type stars.

We have compared our result with the velocity distribution derived by mean of the direct method. Namely, we have taken the F-type stars with known radial-velocity,



sistent with the data derived by old methods. In the case of F-type stars we have some discordances.

It is to be noted here that the symmetrical distribution of the points, where the function $\varphi(U, \beta)$ is determined, around the Sun's velocity is inappropriate for detection of the symmetry and antisymmetry properties in the velocity distribution.

Perhaps it is better to correct radial velocities for solar motion before using them in our method.

We have shown that our method is applicable to the observational data. We have obtained the reliable frequency functions for some types of stars. But the practical side of the method may be doubtless improved further. We hope that these improvements will bring new results in this field.

О РАСПРЕДЕЛЕНИИ ПРОСТРАНСТВЕННЫХ СКОРОСТЕЙ ЗВЕЗД ТИПОВ В И F

В. А. Амбарцумян

(Резюме)

В настоящей работе дается применение разработанного автором метода определения распределения пространственных скоростей из наблюдаемых радиальных скоростей. Именно, выведено распределение проекций пространственных скоростей на галактическую плоскость из рассмотрения радиальных скоростей звезд, лежащих в низких галактических широтах.

Метод применен к трем различным группам звезд (B_0 — B_3 , B_5 — B_9 и F). Результаты применения к звездам типа В оказываются в хорошем согласии со всем тем, что мы уже ранее знали о распределении скоростей этих звезд. Что касается до звезд типа F, то для них получается вторичный максимум функции распределения для скорости 10 км/сек в направлении 90° галактической долготы. Это расходится с данными о полных скоростях звезд этого типа. Численные значения функции распределения, выведенные по радиальным скоростям, даны на фиг. 1, 2 и 3. Каждый из этих графиков представляет плоскость скоростей, точнее проекций скоростей на галактическую плоскость. Центр графиков соответствует скорости Солнца. Следовательно все скорости отнесены к Солнцу.